

## Blaise and Bayes

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"Pascal's Wager" is something of a misnomer. In a single paragraph of his *Pensées*, Blaise Pascal in fact presents *three* wagers—three arguments for believing, or for at least taking steps to believe, in God—as Hacking has observed in his paper "The Logic of Pascal's Wager" (1972). Hacking reconstructs them using the apparatus of Bayesian decision theory. He also makes a point of granting that each of these arguments is *valid*. While he questions Pascal's premises, he is emphatic that Pascal's conclusion really does follow from those premises in each case. I will contend here, equally emphatically, that all three arguments are invalid.

Hacking portrays the first wager as an "argument from dominance", although as we'll see, he is somewhat unclear about what that means. He contends that an argument from dominance is automatically valid: if there is a dominating act, then rationality requires it to be performed. I demur, showing that such an argument is invalid without a further assumption about the relevant conditional probabilities. McLennen characterizes the first wager with a stronger sense of dominance, which he calls "superdominance". But I show that even superdominant acts in his sense need not be performed. He understates the strength of the first wager: it is stronger than one of superdominance in his sense, twice over. So I strengthen his notion of superdominance twice over, finally doing justice to the first wager. And yet *still* the argument is invalid, I argue. I offer two ways that the premises of the argument could be strengthened so as to render it valid, although then the premises are considerably less plausible.

I next turn to Pascal's second and third wagers, which famously appeal to considerations of expected utility. The second wager makes the highly implausible assumption that the probability of God's existence is exactly  $\frac{1}{2}$ , but the third wager weakens this assumption to the probability of God's existence merely being positive—

this is Pascal's best-known "wager". Rehearsing and building on points I made in my 2003 paper "Waging War on Pascal's Wager", I argue that these wagers are also invalid. I then attempt some reparation work. In that paper, I offered four valid reformulations of the wager, although I argued that they did violence to Pascal's theology. Here I will offer two further formulations, and at least the first of these does more justice to that theology.

### **The Argument from Dominance**

Hacking has done us a service in distinguishing the three arguments, and in showing how each can be given a decision-theoretic gloss. I will follow his naming of the arguments, but I will differ from him on some points of detail, and I will part company with him when it comes to assessing them.

The first argument is "the argument from dominance". Pascal maintains that we are incapable of knowing whether God exists or not, yet we must "wager" one way or the other. Reason cannot settle which way we should incline, but a consideration of the relevant outcomes supposedly can:

You have two things to lose, the true and the good; and two things to stake, your reason and your will, your knowledge and your happiness; and your nature has two things to shun, error and misery... Let us weigh the gain and the loss in wagering that God is... If you gain, you gain all; if you lose, you lose nothing. Wager, then, without hesitation that He is.<sup>1</sup>

So we have two possible states of the world: God exists, or God does not exist; and two possible actions: wagering that God exists, or wagering that He does not. It will not be important for our purposes to determine exactly what such "wagerings" involve. Let's simply accept Pascal's idea that wagering for God involves living a pious life, "taking the holy water, having masses said, etc.", steps likely to foster (or to sustain) belief in God; and that to avoid taking such steps is to wager against God.

We can formulate the argument in terms of the following decision matrix:

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<sup>1</sup> All quotes by Pascal are taken from Pascal (1910), p. 233.

	<i>God exists</i>	<i>God does not exist</i>
<i>Wager for God</i>	Salvation	Status quo
<i>Wager against God</i>	Misery	Status quo

The idea is that under either hypothesis about the way the world is—God exists, or does not—the result of wagering for God is a least as good as the result of wagering against God. Thus, according to Pascal, wagering for God is rationally required. Hacking says that "the wager 'God is' dominates the wager 'he is not.'" The decision problem is solved. The argument is valid." (25) Earlier, Hacking explains that dominance occurs "when one course of action is better no matter what the world is like..." (22) This is usually called *strict* dominance.

Firstly, note that we do *not* have a case of such dominance here, for if God does not exist, the outcomes are essentially the same. In Hacking's words: "If God is not, then both courses of action are pretty much on a par" (25). In the jargon, we have instead a case of *weak* dominance.<sup>2</sup>

Hacking writes: "If one act dominates all others, the solution to our decision problem is "*perform the dominant act*."" (22) We see, then, why he regards the argument as valid. But this is too quick: even strictly dominating actions are not necessarily the best ones to perform. Whether they are or not depends on the relevant conditional probabilities of states, given actions. Consider the following case. There are two possible states of the world: you are offered an attractive job, or you are not. There are two possible actions you can perform: you can apply for the job, or not. Given that you are offered the job, you prefer not to have applied: what an honor to be offered the job out of the blue! Given

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<sup>2</sup> Immediately after his explanation of dominance, as quoted in the previous paragraph, Hacking goes on to formulate the notion "[s]chematically". But that turns out to be a formulation of weak dominance, thus something different from what immediately preceded it.

that you are not offered the job, you would prefer not to have applied: you don't like being rejected! You want the job, above all, and you have a decent chance of getting it; but you know that in order to get it, you must apply for it. Here is the decision matrix, with the outcomes ranked from best to worst:

	<i>You are offered job</i>	<i>You are not offered job</i>
<i>Not apply</i>	best	3rd best
<i>Apply</i>	2nd best	worst

*Not applying* strictly dominates *applying*; yet *applying* is clearly the best thing for you to do. Note that we can *a fortiori* make the same point for a weakly dominating act. Change the story so that you are indifferent between applying and not applying in the case that you are offered the job either way. Despite the fact that not applying then weakly dominates applying, applying is still the best thing to do. Punch line: neither weak nor strict dominance is sufficient to make an act choice-worthy.<sup>3</sup>

The states of the world and your actions are not (probabilistically) independent of each other: it is far more likely that you are offered the job given that you apply than given that you do not. Now perhaps in Pascal's argument, the states and actions are independent of each other: "whether God exists or not doesn't (probabilistically) depend on what I do", a friend of dominance reasoning might say. And arguably *if* the states and actions *are* independent of each other, then even weak dominance is sufficient to make an act choice-worthy. (Soon we will see that even that is too quick.) But it's not clear that rationality requires you to regard what you do to be independent of whether God exists or not. Maybe by your lights there is positive dependence between wagering for God, and his existence: maybe God helps people come to believe in Him, in which case your wagering for God provides some evidence that He exists. To be sure, that sort of dependence would only help the Wager. But maybe by your lights there is negative

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<sup>3</sup> This is hardly an original point—for example, one-boxers in Newcomb problem and cooperators in the Prisoner's Dilemma have been saying this for years, although these cases are more controversial.

dependence, which would undermine the dominance reasoning. Pascal would need a further premise that rationality forbids this. This introduces probabilistic considerations, which play absolutely no role in this first argument.<sup>4</sup> The fact that wagering for God weakly dominates, or even strictly dominates, wagering against God is not enough.

More germane, perhaps, is the fact that in a certain specific sense, wagering for God (weakly) *superdominates* wagering against God, according to Pascal. McLennen (1994, p. 118) suggests that

... this is really an argument from a principle of superdominance: each of the outcomes associated with betting on God is at least as good as, or better than, each of the outcomes associated with betting against God. In such a case, we are spared the burdensome business of sorting out notions of probabilistic and causal dependence or independence between our choice of an action and the relevant states...

(I prefix the word "weakly", since the inequality "at least as good as" is weak; the words "or better than" don't add anything.) If wagering superdominates not-wagering like this, one cannot go wrong by wagering: even in the worst case scenario, there is no way that wagering against God could do better.

More germane, perhaps, but still not good enough: (weakly) superdominant acts in McLennen's sense are not automatically *the* best ones to perform, not necessarily rationally mandated. Consider this case: either it is snowing in Siberia, or it is not, a matter of indifference to me; and I am indifferent between eating chocolate and eating strawberries:

	<i>Snow in Siberia</i>	<i>No snow in Siberia</i>
<b>I eat chocolate</b>	Pleasure	Pleasure
<b>I eat strawberries</b>	Pleasure	Pleasure

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<sup>4</sup> This premise might instead invoke the notion of *causal* independence—for example, God's existence is causally independent of my wagering or not.

Each of the outcomes associated with eating chocolate is at least as good as each of the outcomes associated with eating strawberries. So eating chocolate (weakly) superdominates eating strawberries in McLennen’s sense. Yet eating chocolate is not rationally mandated. After all, eating strawberries is just as good—indeed, it (weakly) superdominates eating chocolate! I cannot go wrong with chocolate; but I cannot go wrong with strawberries either.

More germane still is the fact that wagering for God (weakly) superdominates wagering against God in a stronger sense, according to Pascal: each of the outcomes associated with wagering for God is at least as good as each of the outcomes associated with wagering against God *and* in at least one state of the world, wagering for God is *strictly* better than wagering against God.

More germane still, but still not good enough: (weakly) superdominant acts, even in this stronger sense, are still not automatically the best ones to perform. Consider this case: ordinarily I would rather go to the party than stay home. But I know that a somewhat irritating person will make a point of going if I go: he pays close attention to my movements, and he loves to corner me, regaling me endlessly with his philosophical views. Going to a party free of him is my favourite outcome, but his presence there would make me indifferent between going, and staying home. Here is the ranking of the outcomes, with a three-way tie for second best:

	<i>He does not go</i>	<i>He goes</i>
<i>I go</i>	Best	2 <sup>nd</sup> best
<i>I stay home</i>	2 <sup>nd</sup> best	2 <sup>nd</sup> best

Going (weakly) superdominates staying home in this stronger sense. But I am still not rationally compelled to prefer going to staying home. For even though I do not rule out his not going (I assign positive probability to the first column), I *do* rule out his not going given that I go. I am convinced that I will get a second best outcome either way, so I can

just as rationally stay home. Note that we are *not* spared the burdensome business of sorting out notions of probabilistic and causal dependence or independence between our choice of an action and the relevant states, for it is just such dependence that undercuts the superdominant action.

But we can imagine the case for going being still stronger. Suppose that the scenario in which I stay home and he does not go is strictly *worse* than all the others—I vividly picture how much fun the party is, all the more so because the annoying guy isn't there, and I'm missing it!

	<i>He does not go</i>	<i>He goes</i>
<i>I go</i>	Best	2 <sup>nd</sup> best
<i>I stay home</i>	Worst	2 <sup>nd</sup> best

We have started with the previous example's superdominance of going over staying home, and we have strengthened it by making one of the payoffs associated with staying home strictly worse than before—in particular, worse than *any* of the payoffs associated with going to the party.

Stronger, but *still* not strong enough! For consistent with all the assumptions we have made, I might assign probability 1 to his going to the party. In that case, I am certain that column 2 will be realized, and I can rationally realize a second best outcome by staying home: after all, by my lights the best outcome can't be realized.

This case parallels Pascal's first wager. Obviously the payoffs of the outcomes are ordered, with 'salvation' best, 'status quo' second best, and 'misery' worst:

	<i>God exists</i>	<i>God does not exist</i>
<i>Wager for God</i>	Best	2 <sup>nd</sup> best
<i>Wager against God</i>	Worst	2 <sup>nd</sup> best

Both Hacking and McLennen have understated Pascal's case in favor of wagering for God: it doesn't merely dominate wagering against God, in Hacking's sense, and it doesn't merely superdominate wagering against God, in McLennen's sense. It doesn't merely superdominate even in the stronger sense that I introduced (after the words "More germane still ..."), with wagering for God *strictly* better than wagering against God in at least one state of the world. Yet *still* the argument is invalid. A strict atheist who is probabilistically certain that column 2 is realized can rationally realize a second best outcome by wagering against God: after all, by his lights the best outcome can't be realized.

Notice that I said 'probabilistically certain' rather than 'certain': probability 1 does not imply certainty. For example, suppose a fair coin is tossed infinitely many times. It is not *certain* to land heads eventually—it could land tails on every toss. Yet the probability of it landing heads eventually is 1. Probability theorists are not being perverse when they call their various limit theorems 'almost sure' results: the strong law of large numbers, the central limit theorem, and so on. This is not misleading, the way that calling  $\neg(p \ \& \ \neg p)$  'almost a tautology' would be. The relevant limiting behavior is not logically necessary. This raises the interesting question of whether an agent who gives probability 1 to God's non-existence, but who is *not* certain of it, should be moved by this argument from superdominance. Standard expected utility theory says *no*: when multiplying the utility of salvation by probability 0 for God's existence in the expectation formula, the resulting 0 makes no contribution to the expectation, whether it represents sure-atheism, or merely almost-sure-atheism. Yet offhand, this seems to be a strike against standard expected utility theory, for it seems to make a difference to superdominance reasoning whether God's existence is a doxastic possibility or not: if it is, then one ought to believe. In that case I should strengthen the point: if our atheist is a *strict* atheistic, with God's existence not even a doxastic possibility, then superdominance reasoning fails.

I said earlier that a friend of dominance reasoning might assume that the states and actions are completely independent of each other. Ironically, we now have a special case of independence of states from acts! Anything that has probability 1 or 0 is probabilistically independent of *everything*, according to the usual Kolmogorovian/Bayesian construal of independence. So even granting independence of states from acts—which protects against a certain kind specious dominance reasoning such as in the initial job-offer example—is not enough: it had better not be the degenerate case of independence that one gets from assigning probability 1 to one of the states.

In sum, we have come some distance from Hacking's characterization of the argument, every step that we have taken has only improved the case for wagering for God, and *still* the argument for wagering for God is invalid. I must therefore disagree with Hacking's assessment of Pascal's first wager. However, we construe the wager—as an argument from dominance, from (weak) superdominance, or from this stronger form of (weak) superdominance—it is simply invalid. Now to turn it into a valid argument, Pascal could:

1. Insist that each of the outcomes associated with wagering for God is *strictly better than* all of the outcomes associated with wagering against God: the worst outcome associated with wagering for God is strictly better than the best outcome associated with wagering against God. We might call this *superduperdominance* of the former over the latter. Then considerations of utilities alone would carry the day for wagering for God. This modification would require some further argument, however: for we would need to be told why one profits from having a *false* belief in God.<sup>5</sup>

2. Deny that rationality permits atheism: forbid an assignment of probability 0 to God's existence. But one wonders whether this is more rational theology than Pascal should allow himself. How does one square this with his claim that "Reason can decide nothing here"? For now it seems that reason can decide at least *something* here: namely,

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<sup>5</sup> I do not insist that this could not be done.

that certain probability assignments, consistent with the probability calculus and orthodox Bayesianism, are rationally impermissible.

At least here there might be a principled argument that generalizes beyond considerations of theism—and here I digress slightly. (The impatient reader could skip forward to the paragraph that begins “Undigressing”.) For some slightly unorthodox Bayesians impose a further constraint on rational credences: they should never assign probability 0 to any possibilities. The property of zeroing out only logical contradictions is sometimes given the memorably unmnemonic name *regularity*. The norm that is being advocated by these Bayesians, then, may be captured by the slogan: *stay regular!* As I would prefer to say it: *stay open-minded*. “Keep the door open, or at least ajar”, advise Edwards, Lindman and Savage (1963) in their defense of regularity. There are several other arguments for regularity besides this pithy aphorism:

- 1) Irregular agents misrepresent their evidence, treating something that is contingent as if it were impossible.
- 2) Having zeroed out a possibility, an irregular agent could never learn it by conditionalization, the usual Bayesian updating rule.
- 3) The irregular agent is susceptible to making bad decisions—for example, betting at infinite odds against contingent propositions, or giving away bets on such propositions for free. In the case at hand, a strict atheist is prepared to bet his life that God does not exist.
- 4) Related, such an agent is susceptible to a *semi-Dutch Book*: placing a bet that cannot win, and that can lose. In the case at hand, a strict atheist is prepared to pay a dollar for a bet that pays a dollar if God does not exist. There is no possible world in which he wins money, and there *are* possible worlds in which he loses money.

None of these arguments for regularity is decisive:

Against 1): It is *part of* the standard Bayesian model that evidence comes in the form of newly acquired certainties, on which one conditionalizes.

Against 2): The appeal to conditionalization is actually an argument *against* regularity, since the two are incompatible! Also, although it is part of the lore that zero probabilities cannot be raised by conditionalization, that seems false: simply consider a case in which *the evidence acquired was previously assigned probability 0*. In that case, the usual ratio formula assigns no value for any conditional probability, given this evidence—it falls silent. But it is not clear that we should interpret this silence as a verdict that all these conditional probabilities undefined. (It may not be like trying to assign a value to  $1/0$ , for which arithmetic gives the verdict that it is undefined.) In that case, we may be *free* to assign values to some of these conditional probabilities.<sup>6</sup> And having done so, upon learning the evidence, those values become your new unconditional probabilities. For example, suppose you assign probability 0 to a coin landing tails forever; still, you may be *free* to assign a value of 1 to the coin landing tails forever *given* that it lands tails forever. And having done so, upon learning that the coin *does* land tails forever, your unconditional probability for its doing so should become 1.

Against 3) and 4): One should not adopt a naïve betting interpretation of credences. Moreover, the relevant sense of possibility should be *epistemic*—and the possibilities in which an agent faces loss may well be *epistemically impossible*, inconsistent with what she knows.

And against Edwards et al's pithy aphorism, there is this one from Arthur Hays Sulzberger (publisher of the *New York Times* from 1935 to 1961): "I believe in an open mind, but not so open that your brains fall out."

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<sup>6</sup> Thanks here to Aidan Lyon.

Moreover, regularity is incompatible with Kolmogorov's probability calculus, coupled with a plausible assumption about the size of the space of possibilities. After all, plausibly there are uncountably many possibilities, and only countably many of them can be given positive probability in Kolmogorov's sense—see Hájek 2003 for a proof. The Many Gods Objection to Pascal's Wager can be summoned to provide a quick argument against the putative rationality norm of obeying regularity: for each real number  $r$ , there is a possible God whose favorite number is  $r$ . By regularity, each such God should be assigned positive probability. But this contradicts Kolmogorov's probability calculus—yet obedience to this calculus is the standard Bayesian rationality norm.

Undigressing, Pascal's reasoning appears to be powerful because it apparently makes minimal assumptions *about probabilities*—most of the work is done by the *utilities*. This is a familiar point regarding the third, and most famous, of his 'wagers', to which I will turn shortly: it only seems to assume that your probability for God's existence is positive, a very weak assumption. (To be sure, assigning infinite utility to salvation is a very *strong* assumption.) It is tempting to think that his first wager makes even weaker probabilistic assumptions: namely, none at all! I have argued that this is not the case. Holding fixed the original payoffs, to render the argument valid, a probabilistic assumption is needed: that rationality forbids the assignment of probability 0 to God's existence. Whether or not this assumption is plausible, the fact remains that Pascal's first argument is invalid as it stands.

We have introduced probabilities into the discussion. Pascal does so explicitly in his second and third wagers. Let us move on, then, to further invalid arguments.

### **The argument from expectation**

Pascal is aware of the objection that wagering for God may require giving something up, so that if God does not exist, the wagerer-for is worse off than the wagerer-against. No form of dominance argument could then apply. So Pascal now makes two pivotal

moves: he asserts that salvation brings *infinite* reward ("an infinity of an infinitely happy life"); and that the probability of God's existence is 1/2. He then essentially argues that the expectation of wagering for God exceeds that of wagering against God, and hence wagering for God should be preferred. Hacking puts it this way:

In the agnostic's existential situation, the optimal payoff if there is no God is a worldly life. The optimal payoff if there is a God is salvation, of incomparably greater value. Hence, if there is an equal chance of God's existence or nonexistence, the expectation of choosing the pious life exceeds that of choosing the worldly one. The argument from expectation concludes, act so that you will come to believe in God... [The argument] is valid..." (27)

Hacking must be tacitly assuming that salvation is also of incomparably greater value than the result of wagering against God if God exists—otherwise the argument is *clearly* invalid. For it is consistent with what he says here that if there is a God, wagerers-against also enjoy the optimal payoff: God bestows salvation upon wagerers-for and wagerers-against alike. In that case, the expectation of choosing the pious life need not exceed that of not choosing the pious life.<sup>7</sup> Indeed, if the "worldly life" of a wagerer-for is worse than that of a wagerer-against (which was the point of the objection that Pascal responds to, after all), then by a (weak) superdominance argument it would seem that wagering-against is preferable. And if, as in the first argument, the worldly lives of wagerer-for and wagerer-against, in the case that God does not exist, are the same (both "status quo"), then the two expectations are the same, so again there is no reason to prefer wagering for God.

Pascal is frustratingly obscure at this crucial point, and he almost deserves misinterpretation here. But I will grant that he makes the assumption that wagering against God results in some incomparably lesser reward than salvation, presumably misery once again (although a worldly life would also do):

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<sup>7</sup> A little pedantically, I avoid Hacking's phrase "choosing the worldly [life]", since 'the worldly life' is here understood as an outcome, not an action. Suppose the wagerer-against gets something *much worse* than a worldly life if God exists. Then if God exists, one can't choose a worldly life at all: one either gets something much better, or something much worse.

	$p = 1/2$	$p = 1/2$
	<b><i>God exists</i></b>	<b><i>God does not exist</i></b>
<b><i>Wager for God</i></b>	Salvation	Worldly life <sub>1</sub>
<b><i>Wager against God</i></b>	Misery	Worldly life <sub>2</sub>

I distinguish here the two worldly-life outcomes, although this is not important. What *is* important is that salvation is sufficiently better than all the other outcomes.<sup>8</sup> We need not actually assume that salvation constitutes an *infinite* reward in order to make the expectation of wagering for God exceed that of wagering against God. But Pascal unequivocally does make that assumption. So he attaches utilities to the outcomes as follows (where the  $f_i$  are finite numbers<sup>9</sup>):

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<sup>8</sup> Note that Hacking makes a mistake here: "The argument from expectation with an equal probability distribution requires only that salvation, if God is, is more valuable than sinful pleasures, when there is no God". (27) No—to see that this is not so, consider:

	$p = 1/2$	$1-p = 1/2$
	<b><i>God exists</i></b>	<b><i>God does not exist</i></b>
<b><i>Wager for God</i></b>	1	-3
<b><i>Wager against God</i></b>	0	0

The utility of salvation is 1 (there is no question of salvation "if God is not", so I don't understand Hacking's use of a conditional here). This is greater than the 0 utility of sinful pleasures (pursued by the wagerer-against, presumably) when there is no God. Yet the expectation of wagering for God is -1, which is less than the expectation of wagering against God, 0.

<sup>9</sup> In assigning a finite value to 'misery', I am assuming that the misery is not infinitely bad, having utility  $-\infty$ .

	$p = 1/2$	$1-p = 1/2$
	<i>God exists</i>	<i>God does not exist</i>
<i>Wager for God</i>	$\infty$	$f_1$
<i>Wager against God</i>	$f_2$	$f_3$

Now the argument has at least the appearance of validity. The expectation of wagering for God is

$$\infty \cdot 1/2 + f_1 \cdot 1/2 = \infty.$$

This exceeds the expectation of wagering against God, namely,

$$f_2 \cdot 1/2 + f_3 \cdot 1/2 = \text{some finite value.}^{10}$$

So far, so good. Still, I contend that Pascal's conclusion that one must wager for God does not follow. In a nutshell, the main point is that the actions {wager for God, wager against God} do not exhaust the set of possible strategies that one might adopt. Moreover, there are *other* strategies besides wagering for God that have infinite expectation, and Pascal must rule these out if he wants to validly conclude that wagering for God is the unique optimal strategy. I develop this point in the next section.

### **The argument from dominating expectation**

Hacking rejects the argument from expectation because of its "monstrous premiss of equal chance" (27). Said more precisely, what is implausible—yes, perhaps even monstrous—is the assumption that rationality requires one to assign equal probability to God's existence and His non-existence. But note that the probability assignment of 1/2 really plays no role in the argument. Any positive (finite<sup>11</sup>) assignment would work

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<sup>10</sup> The calculations will work even if you think that what you do is not independent of whether God exists —just replace the unconditional probabilities by the appropriate conditional probabilities (assuming that they are non-zero).

<sup>11</sup> I discuss the possibility of infinitesimal positive assignments in my (1997).

equally well. So for the third argument, Pascal keeps the same decision matrix, but now essentially allows  $p$  to be a variable, ranging over all positive values:

	$p > 0$	$1 - p < 1$
	<b><i>God exists</i></b>	<b><i>God does not exist</i></b>
<b><i>Wager for God</i></b>	$\infty$	$f_1$
<b><i>Wager against God</i></b>	$f_2$	$f_3$

The argument now runs much as before. The expectation of wagering for God is

$$\infty \cdot p + f_1 \cdot (1 - p) = \infty.$$

This exceeds the expectation of wagering against God, namely,

$$f_2 \cdot p + f_3 \cdot (1 - p) = \text{some finite value.}$$

Now it is supposed to follow from all this, of course, that rationality requires wagering for God. The conclusion does *not* follow, for several reasons. I address them in increasing order of importance.

Firstly, we need at least the further premise that rationality requires one to perform an action of maximal expectation. Now *perhaps* this is an analytic truth, and so can be tacitly assumed—perhaps it is *constitutive* of rationality to maximize expectation, as some would say. But this premise has met serious objections. The Allais and Ellsberg paradoxes, for example, are said to show that maximizing expectation can lead one to perform intuitively sub-optimal actions. So too the St Petersburg paradox, in which it is supposedly absurd that one should be prepared to pay any finite amount to play a game with infinite expectation. (That paradox is particularly apposite here.) Moreover, the usual rationale for maximizing expectation comes from the laws of large numbers. Their content is roughly that under suitable circumstances, in the limit, one's average reward tends to the expectation; and of course one wants to maximize one's average reward. But

being limit theorems, these are results concerning an infinitely long run of trials. Far from having such a long run here, we have just a *single-shot* decision problem. This is a decision that you don't get to repeat. Worse, the *variance* of wagering for God is infinite. Generally, the greater the variance, the shakier is the inference from what one should do in a long run of repeated trials, to what one should do on a single trial. When the variance is infinite, the inference is particularly shaky.

Secondly, can we really make sense of the expectation calculation for wagering for God, in which  $\infty$  is treated as if it were a number, amenable to the relevant arithmetic operations? The very notion of infinite utility is already problematic (see Jeffrey (1983) and McLennen (1994) for skepticism). I am sympathetic to the notion (see my 2003), but I would prefer to see the 'indeterminate' form of infinity,  $\infty$ , replaced by a precise non-standard infinite number, in the style of Robinson (1966) or Conway (1976), so that the operations of multiplication by  $p$ , addition, and so on are straightforward. As the argument stands, I am not sure that expressions such as " $\infty \cdot p$ " are well formed. But I will concede for the sake of argument that they are, for worse news is ahead.

Finally, we come to the main problem with Pascal's reasoning, first pointed out by Duff (1986), and developed further in my (2003). Pascal's specious step is to assume that *only* the strategy of wagering for God gets the infinite expected utility. To see that this is not the case, consider the following strategy: you toss a fair coin, and wager for God iff the coin lands heads (probability one half).<sup>12</sup> What is its expected value? Well, half times infinity is still infinity, and the other terms in the expectation sum are all finite—so we have found another strategy of infinite expected value. Now that we see the trick, we can run it again and again. Wager for God iff a die lands 6 (a sixth times infinity is still infinity ...); iff a pair of dice land double 6; iff your lottery ticket wins next week; iff you see a meteor quantum-tunnel its way through the side of a mountain and come out the

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<sup>12</sup> Is this a genuine strategy? By Pascal's lights, surely: if wagering for God is something you can choose, then so is wagering for God iff the coin lands heads.

other side... Pascal has ignored all these *mixed strategies*, with their various positive weights, and infinitely many more besides—and all of them have the same expectation as the strategy that he favors.

But this still understates Pascal's troubles. For isn't *anything* that an agent might choose to do really a mixed strategy between wagering for and against God, for some appropriate (rational subjective) probability weights? For whatever one does, one should assign some positive probability to winding up wagering for God. Even if you are currently an atheist [dear reader] you should assign positive probability to your wagering for God by the time I reach the end of this sentence (a probability greater by many orders of magnitude, I would hazard to say, than the probability of the meteor tunneling). In fact, I would hazard to say that *every* rational agent's life is a constant series of such 'gambles', with wagering for God as one of the payoffs. This is so even for those who single-mindedly do all they can to *avoid* wagering for God—by practising devil worship, say. They all nonetheless should have a positive probability of wagering for God at the end of any course of action. And that means that by Pascal's lights, everybody enjoys maximal expected utility at all times. It seems that we have here a 'proof' that Leibniz was on the right track after all: in an important sense, this really is the best of all possible worlds!<sup>13</sup>

So I must part company with Hacking: again, Pascal has given an invalid argument. Now, there are several ways that Pascal could reformulate it so as to produce valid arguments. In my (2003), I considered four:

1. Replace the  $\infty$  utility for salvation with some enormous finite value. In particular, make that value so large that it offsets even the smallest positive probability for God's existence that any human ever had, or ever will have.

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<sup>13</sup> The sense being that there is no inhabitant of the actual world who has better expectation in another possible world.

2. Replace the  $\infty$  utility for salvation with a non-standard infinite valued utility—for example  $\omega$  in Conway's surreal number system.

3. Replace the utilities in Pascal's decision matrix with two-dimensional, lexically ordered, vector utilities; stipulate that salvation receives a utility that lexically dominates all utilities gained in a worldly life.

4. Regard infinite utility as the total utility achieved by having a finite good for an infinite period of time. Then apply the economist's notion of the *long-run average* to distinguish among such infinite utilities: calculate the total utility up to time  $t$ , divide by  $t$ , then take the limit as  $t$  tends to  $\infty$ .

Now I want to consider two more ways to render the argument valid.

### 5. Salvation has infinite utility, damnation has negative infinite utility

The key here will avowedly be something of a mathematical trick—but it seems that the trick *does* work.

It is part of various theologies that if God exists, the non-believer is damned. We might take this to mean infinitely awful punishment, or perhaps finitely awful punishment for infinite time without discounting for the future. The thought is that we should represent the relevant cell of the utility matrix to be  $-\infty$ . Here's the matrix:

	God exists	God does not exist
Wager for God	$\infty$	$f_1$
Wager against God	$-\infty$	$f_2$

Wagering for God has infinite expectation, wagering against God has negative infinite expectation. Interestingly, any mixed strategy between wagering-for and wagering-against has expectation  $\infty - \infty$ , which is undefined, or as Jeffrey says, "indeterminate" (154).<sup>14</sup>

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<sup>14</sup> Sobel (1996) gives an argument for judging this sum to be "not defined" (37).

Still, it seems that we can reach a verdict as to what should be done. Given a choice between an act whose expected utility is infinite, and an act whose expected utility is indeterminate, the former is preferable. It's hard to know how to think about indeterminate expected utility, but it is certainly no *better* than infinite, and it does seem worse. The choice between  $\infty$  and  $\infty - \infty$  seems clear: wagering for God uniquely maximizes your expectation.

### 6. Salvation has finite utility, damnation has negative infinite utility

This indeterminacy will go away if we remove one of the infinities. Pascal removed the negative infinity, and previously I have done so too. But now let us remove the positive infinite utility instead.

The proposal, then, is to replace Pascal's decision matrix by:

	God exists	God does not exist
Wager for God	$f_1$	$f_2$
Wager against God	$-\infty$	$f_3$

Here, a concern to gain an infinite reward is replaced with a concern to avoid infinite loss of utility. Presumably  $f_2$  and  $f_3$  are smaller than  $f_1$ ; but this is not essential to the argument. Whatever they are, the expectation of wagering for God is finite, and of wagering against God is negative infinite, under our assumption of positive probability for God's existence. Moreover, wagering for God, with its finite expectation, is the unique optimum, so once again we can validly conclude that rationality requires you to wager for God.

We could replace the utility of  $-\infty$  for damnation much as we replaced the utility for salvation in the first four reformulations, generating still further hybrid variants of the

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Wager: one with enormous but finite negative utility, another with a surreal negative infinite utility, another with two dimensions of value (with heavenly value now going as low as  $-1$ ), and another with a finite negative utility for infinite time. We could make any of these replacements, but we need not—the argument is valid without finessing.

However, it's worth noting that Robinson-style non-standard analysis has no particular difficulties with handling infinity minus infinity. There need be no indefiniteness in infinity minus infinity. If  $J_1$  and  $J_2$  are both positive infinities, then  $J_1 - J_2$  is perfectly well defined. For some choices of  $J_1$  and  $J_2$ , it'll be itself a positive infinity; for other choices it'll be a negative infinity; for other choices it'll be zero, or a finite non-zero number, or even an infinitesimal.

Note that variants 5 and 6 distinguish in expectation outright wagering for God, but lump together all the mixed strategies (and this is another respect in which they differ from the first four reformulations). For all the mixed strategies are indeterminate in expectation in 5, and have expectation  $-\infty$  in 6. 5 at least distinguishes the mixed strategies from outright wagering against God (indeterminate expectation, as opposed to  $-\infty$ ). 6 doesn't even make that distinction. So if one gives positive probability to winding up wagering against God (because one is playing a mixed strategy, or to preserve regularity, or for some other reason), then according to 6 this is just as bad as outright wagering against God.

6 is surely incompatible with Pascal's theology—he writes, for example, of salvation as “an infinity of infinitely happy life”. Arguably, even 5 is incompatible with Pascal's theology. He writes: ““The justice of God must be vast like His compassion. Now justice to the outcast is less vast ... than mercy towards the elect”. This supports the reading of the final Wager as assigning infinite utility to salvation, and only *finite* disutility to damnation. So I do not believe that he gave us a Wager with 5 as its decision matrix, although it is a common interpretation of the Wager. But whether or not it is congenial to his theology, at least it has the virtue of being valid.

In any case, all three of the Wagers that Pascal *did* give us are, *pace* Hacking, invalid as they stand.<sup>15</sup>

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